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NOTE FOR THE PAPER: A PROPOSAL OF THE SCALING RELATION FOR THE SHORT-PERIOD LEVEL ASSOCIATED WITH THE THREE-STAGE MODEL OF INLAND CRUSTAL EARTHQUAKES

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ABSTRACT: To solve the problem of evaluating the area of SMGA data S_{SMGA}, which was a problem in an examination of the strong motion generation area (SMGA) data in the previous paper (Tohdo et al., 2022), the area S_{SMGA} was evaluated directly from SMGA data in a mean sense. In this evaluation, S_{SMGA} was scaled with $M_0^{2/3}$, $M_0^{1/2}$, and M_0^1 for seismic moment M_0, associated with the three-stage model in the "Recipe" by HERP. The empirical equation for the relationship of M_0–S_{SMGA} was established by applying the least-squares method to the SMGA data, and the stress drop calculated using this equation and the pre-set empirical equation for the short-period level was constant regardless of M_0 for each stage of the three-stage, and it was confirmed that they were consistent with the SMGA data.

Keywords: *Three-stage model, Strong motion generation area, SMGA-area, Empirical equation*

1. INTRODUCTION

In the previous paper[1)], we proposed the three-fold-line model [Eq. (10) in the previous paper] [1)] for the relationship between the seismic moment M_0 and the short-period level A, which is the scaling relation associated with the M_0–S scaling law on the fault area S by the three-stage model for inland crustal earthquakes in the "Recipe"[2] published by the Headquarters for Earthquake Research Promotion (HERP). The three-fold-line model can represent the characteristics of earthquake observation records, so that the ratio S_{asp}/S of asperity area S_{asp} to fault area S, and stress drop on asperity $\Delta\sigma_{asp}$ are independent of M_0.

To verify the validity of the relationship of M_0–A by the three-fold-line model, we collected data on the area S_{SMGA} and stress drop $\Delta\sigma_{SMGA}$ of the strong motion generation area (SMGA), which is considered equivalent to the parameters (S_{asp}, $\Delta\sigma_{asp}$) of the asperity model. We also collected data of fault area S based on the heterogeneous slip distribution of the source inversion results. These data on SMGA from 16 earthquakes were listed in Table 1 of the previous paper [1]. The geometric means of the area ratio S_{SMGA}/S and the stress drop $\Delta\sigma_{SMGA}$ obtained from the collected SMGA data were 0.183 and 14.0 MPa, respectively. In Section 4.3 of the previous paper[1], as one of the studies on the validity of the three-fold-line model, we obtained the empirical equation for the relationship of M_0–A_{SMGA} from the short-period level A_{SMGA} of SMGA data [Eq. (13) in the previous paper] [1], which corresponds well with the three-fold-line model. In Section 4.3 [1], then, the stress drop of the SMGA $\Delta\sigma_{SMGA}$ was calculated by the theoretical relation [Eq. (11)][1] using the empirical equation of M_0–A_{SMGA} [Eq. (13)][1] and the area ratio S_{SMGA}/S of 0.183. As the result, the stress drops $\Delta\sigma_{SMGA}$ were 15.5 MPa, 16.1 MPa, and 16.1 MPa for each stage, however they were not consistent with the geometric mean $\Delta\sigma_{SMGA}$ of 14.0 MPa from the SMGA data. This problem was considered to be caused by the fact that the area obtained by multiplying the fault area S of the M_0–S relation [Eq. (1) or Eq. (2)][1] in the three-stage model of the Recipe with 0.183 used to calculate $\Delta\sigma_{SMGA}$ does not directly represent the area of the collected SMGA data S_{SMGA}. Therefore, it was a problem to appropriately evaluate the area of the SMGA data S_{SMGA} [listed in Table 1 of the previous paper] [1].

In this study, to solve the above problem, we set an empirical equation for the relationship of M_0–S_{SMGA} from the SMGA data, which is associated with the three-stage model in the Recipe, indicating the area of the SMGA data S_{SMGA} in a mean sense. Next, the empirical equations for the relationship of M_0–S_{SMGA} and the relationship of M_0–A_{SMGA} are substituted into the theoretical equation for the short-period level to obtain the stress drop. These are compared with the stress drop of the SMGA data $\Delta\sigma_{SMGA}$. This study complements Section 4.3 of the previous paper [1].

2. FORMULATION OF THE SCALING LAW OF THE M_0– S_{ASP} RELATIONSHIP

First, to formulate the scaling relationship for the area of asperity S_{asp} with respect to M_0 based on the relationship of M_0–S [Eq. (1) or Eq. (2)][1] of the three-stage model in the Recipe, it is assumed that the area ratio S_{asp}/S can be expressed as a constant regardless of M_0. Based on this assumption, when the S value of the relationship of M_0–S is multiplied by a constant area ratio, the asperity area S_{asp} at each stage can be expressed using Eq. (1), with scales of $M_0^{2/3}$, $M_0^{1/2}$, and M_0^{1}: The range of M_0 (Nm) for each stage is the same as that mentioned in the three-stage model of the Recipe:

$$S_{asp} = \begin{cases} \eta_1 \times M_0^{2/3} & , \quad M_0 < 7.5\times10^{18} \\ \eta_2 \times M_0^{1/2} & , \quad 7.5\times10^{18} \le M_0 \le 1.8\times10^{20} \\ \eta_3 \times M_0 & , \quad M_0 > 1.8\times10^{20} \end{cases} \tag{1}$$

Here, η_1, η_2 and η_3 are constants for each stage.

Tajima et al. [3] and Miyakoshi et al. [4] have shown that the area of the SMGA almost coincides with the area of asperity, as pointed out by Miyake et al. [5]. Based on these previous studies, the relationship of Eq. (1) is applied to the relationship of M_0–S_{SMGA} in the examination of the area of SMGA data S_{SMGA} in Section 3.

3. SETTING AN EMPIRICAL EQUATION FOR THE AREA OF THE SMGA DATA AND EXAMINING STRESS DROP

Next, we set the same scaling relationship of M_0–S_{SMGA} as in Eq. (1) and determined the constants of the relationship by the least-squares method using the area of the SMGA data S_{SMGA} of the collected 16 earthquakes [listed in Table 1 of the previous paper] [1)]. Considering that the area S of the three-stage model in the Recipe [Eq. (1) or Eq. (2)][1)] is not continuous at the boundary of $M_0 = 7.5\times10^{18}$ Nm between the first stage and the second stage, the ratio of 0.926 of the second stage to the first stage for S at $M_0 = 7.5\times10^{18}$ Nm was given as the ratio of the area S_{SMGA} at the boundary of $M_0 = 7.5\times10^{18}$ Nm according to the first and second statements of Eq. (1). Equation (2) is an empirical equation for the relationship of M_0–S_{SMGA} between the seismic moment (M_0; Nm) and the area of the SMGA (S_{SMGA}; km^2), obtained by the least-squares method using S_{SMGA} data of 16 earthquakes [Table 1] [1)].

$$S_{SMGA} = \begin{cases} 2.37\times10^{-11}\times M_0^{2/3} & , \quad M_0 < 7.5\times10^{18} \\ 3.07\times10^{-8}\times M_0^{1/2} & , \quad 7.5\times10^{18} \le M_0 \le 1.8\times10^{20} \\ 2.29\times10^{-18}\times M_0 & , \quad M_0 > 1.8\times10^{20} \end{cases} \tag{2}$$

Figure 1 depicts the comparison of the M_0–S_{SMGA} relationship of the SMGA data and that according to the empirical equation of Eq. (2) shown by the thick solid line. The natural logarithmic standard deviation for error estimated by Eq. (2) from 16 SMGA data was 0.42. Equation (2) corresponds well to the S_{SMGA} data in the range from the first to third stages. By comparing the relationship of M_0–S_{SMGA} in Eq. (2) with the relationship of M_0–S of the three-stage model in the Recipe [Eq. (1) or Eq. (2)][1)], the ratio of S_{SMGA} to S at any M_0 was 0.229, which is the same from the first to the third stages. In other words, the area S_{SMGA} equivalent to that expressed by Eq. (2) can be obtained by multiplying the fault area S of the three-stage model with a constant value of 0.229 regardless of M_0.

The thin solid line in Fig. 1 shows the relationship of M_0–S_{asp} by the empirical equation in which Somerville et al.[6)] scaled S_{asp} by $M_0^{2/3}$ from the slip distribution of source inversion results. The ratio on the constant term of the statement in the first stage of Eq. (2) to that of the empirical equation by Somerville et al. was 1.02. From this result, it can be seen that the relationship of M_0–S_{SMGA} in the first

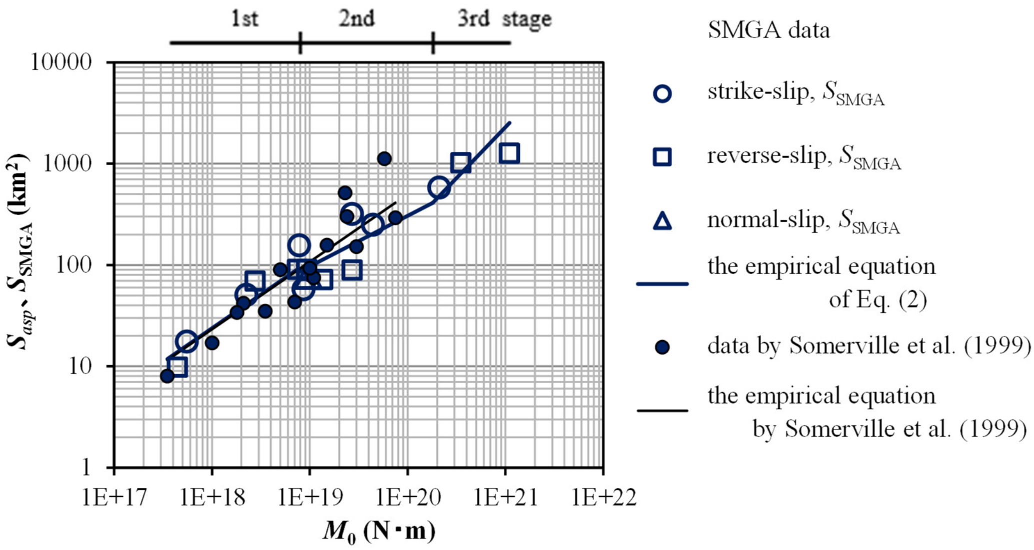

Fig. 1 M_0–S_{SMGA} relationship between the seismic moment M_0 and area of the SMGA data S_{SMGA}. The thick solid lines are the relationship of M_0–S_{SMGA} from the empirical equation of Eq. (2). The thin solid line is the relationship of M_0–S_{asp} from the empirical equation by Somerville et al.[6)], and the solid circles are their data used.

stage of Eq. (2) has the same scaling of $M_0^{2/3}$ as the relationship of M_0–S_{asp} on asperity by Somerville et al.[6], and as pointed out by Miyake et al. [5], S_{SMGA} and S_{asp} are equivalent.

As for the short-period level A_{SMGA}, the empirical equation for the relationship of M_0–A_{SMGA} has been obtained using the same SMGA data [listed in Table 1 of the previous paper] [1], that is shown in Eq. (3) [repost Eq. (13) of the previous paper[1]] :

$$A_{SMGA} = \begin{cases} 5.73\times10^{12}\times M_0^{1/3} & , \quad M_0 < 7.5\times10^{18} \\ 2.14\times10^{14}\times M_0^{1/4} & , \quad 7.5\times10^{18} \le M_0 \le 1.8\times10^{20} \\ 1.85\times10^{9}\times M_0^{1/2} & , \quad M_0 > 1.8\times10^{20} \end{cases} \tag{3}$$

where A_{SMGA} is in Nm/s^2 and M_0 is in Nm. The natural logarithmic standard deviation of the ratios of A_{SMGA} of the SMGA data to the estimated A_{SMGA} by substituting M_0 into Eq. (3) from 16 SMGA data was 0.23.

Since the empirical equation for the relationship of M_0–S_{SMGA} by Eq. (2) was established, in addition to the empirical equation for the relationship of M_0–A_{SMGA} by Eq. (3), both of which express the SMGA data in a mean sense, using both equations the stress drop $\Delta\sigma_{SMGA}$ is calculated by Eq. (4) based on the theoretical equation for the short-period level [Eq. (4)] [1].

$$\Delta\sigma_{SMGA} = \frac{A_{SMGA}}{4\pi\cdot\beta^2\cdot\sqrt{S_{SMGA}/\pi}} \tag{4}$$

Substituting Eq. (2), Eq. (3) and the S-wave velocity β of 3.46km/s[1] into Eq. (4), the stress drops of SMGA $\Delta\sigma_{SMGA}$ in the first, second, and third stages become 13.9 MPa, 14.4 MPa, and 14.4 MPa, respectively, regardless of M_0. The stress drop $\Delta\sigma_{SMGA}$ estimated from these empirical equations and $\Delta\sigma_{SMGA}$ of the SMGA data were compared, as shown in Fig. 2. The solid lines in Fig. 2 are $\Delta\sigma_{SMGA}$ calculated by Eq. (4) using both empirical equations for A_{SMGA} and S_{SMGA}; the dashed lines are $\Delta\sigma_{SMGA}$ using the empirical equation for A_{SMGA} by Eq. (3) and the area ratio S_{SMGA}/S of 0.183 [shown in Fig. 8 of

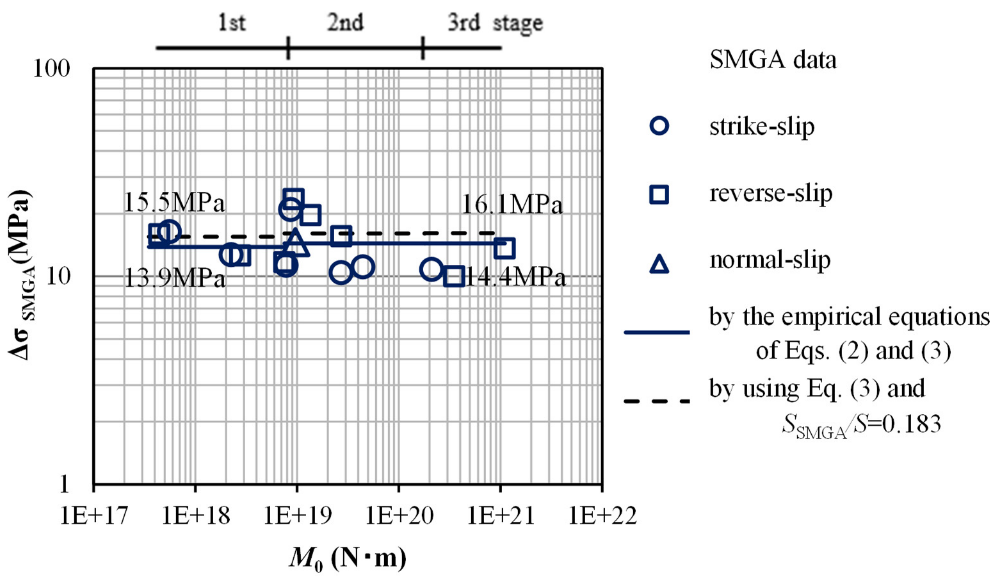

Fig. 2 M_0–$\Delta\sigma_{SMGA}$ relationship between the seismic moment M_0 and stress drop of the SMGA data $\Delta\sigma_{SMGA}$. The solid lines are the relationship of M_0–$\Delta\sigma_{SMGA}$ calculated by Eq. (4) using both empirical equations for S_{SMGA} in Eq. (2) and A_{SMGA} in Eq. (3), and the dashed lines are the M_0–$\Delta\sigma_{SMGA}$ using A_{SMGA} in Eq. (3) and the area ratio S_{SMGA}/S of 0.183.

the previous paper] [1].

Comparing the stress drop $\Delta\sigma_{SMGA}$ of the SMGA data with the $\Delta\sigma_{SMGA}$ calculated from Eq. (4), which was obtained by applying both empirical equations for S_{SMGA} in Eq. (2) and A_{SMGA} in Eq. (3), we found that the stress drops $\Delta\sigma_{SMGA}$ calculated by Eq. (4) using the empirical equations based on the SMGA data are almost consistent with the $\Delta\sigma_{SMGA}$ of the SMGA data with the geometric mean of 14.0 MPa from the 16 earthquakes [listed in Table 1] [1].

4. CONCLUSIONS

We examined the problem on evaluating the area of SMGA data S_{SMGA} in Section 4.3 of the previous paper[1]. In this study, the area S_{SMGA} was scaled with $M_0^{2/3}$, $M_0^{1/2}$, and M_0^1 for seismic moment M_0, associated with the three-stage model in the Recipe. Then, the empirical equation of Eq. (2) for the relationship of M_0–S_{SMGA} was established using the least-squares method to the SMGA data in a mean sense. The area S_{SMGA} equivalent to that expressed by Eq. (2) can be obtained by multiplying the fault area S of the three-stage model in the Recipe with a constant value of 0.229.

Applying the empirical equations for the relationship of M_0–S_{SMGA} in Eq. (2) and the relationship of M_0–A_{SMGA} for the short-period level A_{SMGA} in Eq. (3) obtained from the SMGA data, the stress drops of SMGA $\Delta\sigma_{SMGA}$ in the first, second, and third stages become 13.9 MPa, 14.4 MPa, and 14.4 MPa, respectively, regardless of M_0. These stress drops were almost consistent with the $\Delta\sigma_{SMGA}$ of the SMGA data, the geometric mean of which was 14.0 MPa.

In this study, we considered that the area of asperity and the area of SMGA are equivalent. However, the periodic ranges and analysis methods used for extraction of the asperity and SMGA from a source are different between them. Therefore, the degree of consistency of the characteristics including the number and arrangement of asperity and SMGA within a source is an issue to be examined in the future.

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REFERENCES

1) Tohdo, M., Arai, K., Miyakoshi, J., Sato, T., Fujiwara, H. and Morikawa, N.: A Proposal of the Scaling Relation for the Short-Period Level Associated with the Three-Stage Model of Inland Crustal Earthquakes, *Journal of Japan Association for Earthquake Engineering*, Vol. 22, No. 5, pp. 43–59, 2022 (in Japanese with English abstract).
2) Headquarters for Earthquake Research Promotion: Strong Ground Motion Prediction Method for Earthquakes with Specified Source Faults ("Recipe"), 2020 (in Japanese). https://www.jishin.go.jp/main/chousa/20_yosokuchizu/recipe.pdf (last accessed on January 6, 2022)
3) Tajima, R., Matsumoto, Y., Si, H. and Irikura, K.: Comparative Study on Scaling Relations of Source Parameters for Great Earthquakes in Inland Crusts and on Subducting Plate-Boundaries, *Zisin* (*Journal of the Seismological Society of Japan*), Vol. 66, pp. 31–45, 2013 (in Japanese with English abstract).
4) Miyakoshi, K., Irikura, K. and Kamae, K.: Re-Examination of Scaling Relationships of Source Parameters of the Inland Crustal Earthquakes in Japan Based on the Waveform Inversion of Strong Motion Data, *Journal of Japan Association for Earthquake Engineering*, Vol. 15, No. 7, pp. 141–

156, 2015 (in Japanese with English abstract).
5) Miyake, H., Iwata, T. and Irikura, K.: Source Characterization for Broadband Ground-Motion Simulation: Kinematic Heterogeneous Source Model and Strong Motion Generation Area, *Bulletin of the Seismological Society of America*, Vol. 93, pp. 2531–2545, 2003.
6) Somerville, P. G., Irikura, K., Graves, R., Sawada, S., Wald, D., Abrahamson, N., Iwasaki, Y., Kagawa, T., Smith, N. and Kowada, A.: Characterizing Crustal Earthquake Slip Models for the Prediction of Strong Ground Motion, *Seismological Research Letters*, Vol. 70, No. 1, pp. 59–80, 1999.

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